WILLIAM & MARY
CHARTERED 1693
Training & Technical Assistance Center
P.O. Box 8795
Williamsburg, VA 23187-8795

Geometry Strategies for Middle School Considerations Packet

For more information contact:

E-mail: ttacwm@wm.edu
Phone: 757-221-6000 or 800-323-4489
Website: http://education.wm.edu/centers/ttac/index.php

Geometry Strategies for Middle School

This *Considerations Packet* describes strategies middle school mathematics teachers can incorporate into their teaching of geometry. An overview of the van Hiele Model is followed by a description of how to assess students' level. Strategies for teaching plane figures, perimeter and area, geometric solids, and transformations are also included.

The van Hiele Levels

During the middle school years, most students are making the transition from inductive methods of reasoning (conclusions based on several past observations) to a more formal method of deductive reasoning (proving statements from accepted postulates, definitions, theorems and given information) (National Council of Teachers of Mathematics, 2000). However, students at this age may be functioning at many different levels of reasoning ability. Therefore, before beginning instruction, it is important to assess students' reasoning level. This allows teachers to differentiate instruction based on student readiness.

There are five developmental levels of geometric reasoning based on a study by Dina van Hiele-Geldof and her husband, Pierre Marie van Hiele. They are:

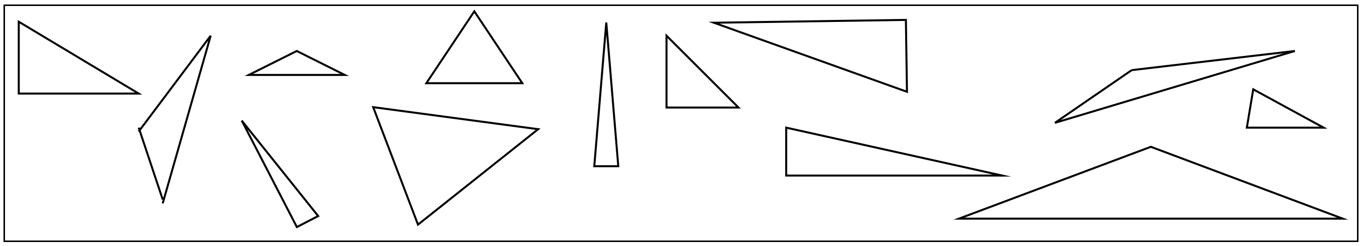
- Level 0 (Basic Level): Visualization
  At this level students view objects as entire entities, not noticing individual components or properties. The focus is on the whole object, not its parts.
- Level 1: Analysis
  Students begin to recognize that geometric shapes have parts and special properties. However, they are not able to describe how these properties are related, nor are they able to understand definitions.
- Level 2: Informal Deduction
  At this level students comprehend the connection between properties within geometric figures and from one set of figures to another. Students are able to follow proofs, but are not able to construct one themselves.
- Level 3: Deduction
  At this level students can construct a geometric proof and understand the connection between postulates, theorems, and undefined terms.
- Level 4: Rigor
  At this level students see geometry in the abstract. Students can move between different geometric systems and can compare and contrast them (Crowley, 1987).

A number of assumptions are basic to the van Hiele model.

- Students' levels are not affected by their age.
- Students must master each developmental level to progress in their geometric understanding.
- Level is determined by concepts that have been taught to the students.

At the middle school level it is of utmost importance that teachers begin to prepare students for the more formal study of geometry to follow in high school. Most high school courses are taught with the understanding that students are reasoning at level 3 or 4. Therefore, it is the responsibility of the middle school teacher to move students in that direction (NCTM, 2000).

In order for teachers to identify the developmental level or geometric reasoning of each of their students, assessment is required. The following activity serves as a tool to determine each student's geometric reasoning level, which will subsequently inform instruction. Using the draw feature in a word processing program, create a handout that consists of at least 20 triangles of varying sizes and classifications. For example:



Have students sort the triangles into as many sets as possible. Then ask students to write a paragraph describing why they placed each triangle into a given set. Using this information, the teacher should be able to determine students' developmental level based on the following divisions:

- Level 0: Students divide the triangles into sets based on size (i.e., small, medium, and large).
- Level 1: Students divide the triangles according to one characteristic, most likely focusing on either the length of sides or the size of the angles.
- Level 2: Students observe more than one characteristic of the triangles. For example, they will see that there are isosceles right triangles and scalene right triangles, or that an isosceles triangle can be right, acute, or obtuse.
- Level 3: Students use definitions, postulates, or theorems to make connections and to reason.
- Level 4: Students grasp abstract concepts and apply them through more than one geometric system.

Students at the middle school level are usually moving towards level 3 with few to none having reached level 4 (NCTM, 2000).

Once the developmental levels of the students in the class have been determined, the teacher is able to determine the students' instructional level. Students taught at their instructional level master skills necessary for them to progress in geometric reasoning. If a student is at one developmental level and the teacher instructs concepts at a different developmental level, it is very likely that the student will not grasp and retain the information (Crowley, 1987).

Strategies to Teach Plane Figures

The following strategies have been used effectively to teach plane figures to middle school students. Research has shown that when teachers incorporate these four strategies in their instruction, retention is increased (Marzano, Pickering, & Pollock, 2001).These strategies include:

- Manipulatives such as geo-strips or fasteners and tag board
- Cooperative learning using the jigsaw method
- Similarities and differences using Venn diagrams

- Vocabulary enhancement using a manipulative and group activity

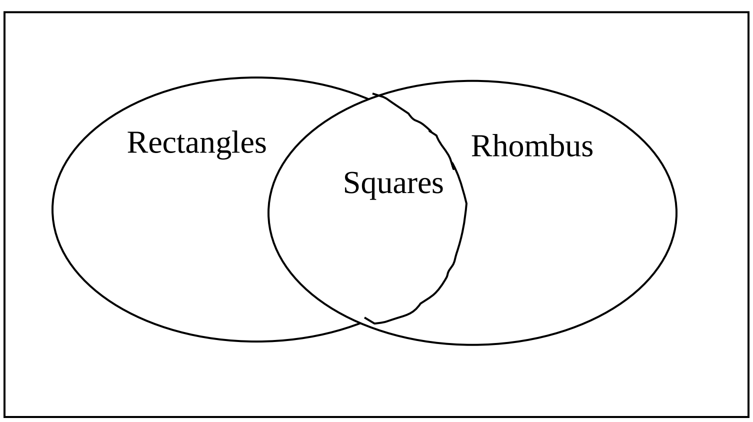
Manipulatives
When parallelograms are first introduced to the class, it is helpful for the students to have a manipulative to explore. Geo strips, which can be made of varying size strips of tag board and brat fasteners, help students discover the properties of parallelograms.

Jigsaw Method
Once students have been introduced to parallelograms and their basic properties, the jigsaw method may be used to further explore special types of quadrilaterals. Teachers can follow these steps:

1. Divide the class into groups of four. Within each group assign a student to be a rectangle, square, rhombus, or trapezoid.
2. The "expert" from each group will leave their home group and meet together with the experts from the other teams. For example, all the rectangles will meet in one corner, the rhombi in another, and so on.
3. Provide each group with a guided activity that will allow members to explore their shape and learn its properties. The group members must come to a consensus on the properties and feel confident that they can teach these properties to their home teams.
4. The " expert" group for each figure should prepare examples, diagrams, properties, and three quiz questions to share with their home teams.
5. After the allotted time, students return to their home teams to share their knowledge with their respective groups (Posamentier, Hartman, & Kaiser, 1998).

Venn Diagrams
As students further study the properties of different types of parallelograms, they need to learn how to compare and contrast the properties of these shapes. Venn diagrams are an excellent method for displaying the shared as well as unique properties of each type of parallelogram (Marzano et al., 2001).

Venn Diagram Comparing Parallelograms



Vocabulary Enhancement
Finally, to reinforce new vocabulary explored in the unit, students can participate in a group game that focuses on the properties of each quadrilateral. The teacher can do the following:

1. Divide the class into groups of four students.
2. Provide each group with a “construction bag” containing items such as straws, toothpicks, tiles to show right angles, and play dough.
3. Provide each student with a card that contains the description of one of the quadrilaterals studied.
4. Each student must use the items in the bag to construct the quadrilateral on his or her card, making it identifiable to others in his or her group.
5. Using their definitions, students must justify the construction of the figures (NCTM, 2000).

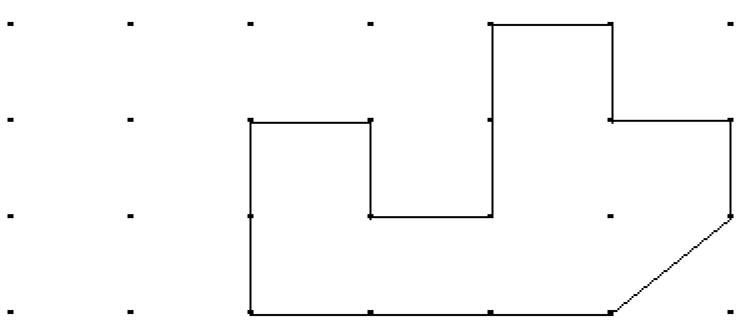
Strategies to Teach Perimeter and Area

Middle school students need concrete experiences with the concepts of perimeter and area as a foundation for future study of these topics in high school geometry. Beginning with manipulatives, moving to pictures, and ending with the formulas enables students to comprehend the meaning of perimeter and area and the differences between the two (Malloy, 1999). Teachers can use the following activities to help students gain a full understanding of these topics:

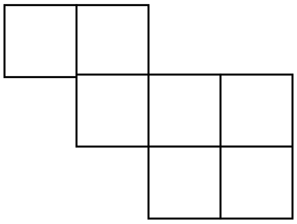
- Area and perimeter using manipulatives
- Index cards to show relationships between formulas

Manipulatives
As the van Hiele research indicates (Crowley, 1987), it is necessary to know students’ developmental levels in order to design appropriate instruction. When working with perimeter and area, the first experience students should have is solving problems to gain information (Malloy, 1999). Teachers should provide students the following:

- Definitions of perimeter and area, but not formulas
- Rectangles and squares on dot paper that can be used to determine the perimeter and area
- Triangles on dot paper to determine perimeter and area
- Irregular shapes such as those shown below to find perimeter and area



Next, provide time for students to interact with the information. Using tiles in the formation below, students may be asked to add tiles to create a perimeter of 18.



Allow students to start out by working independently writing down their process. Ask questions such as the following:

1. Is there more than one possible answer?
2. What is the least number of tiles that can be added? The greatest?
3. What are some strategies that were used to arrive at a solution?

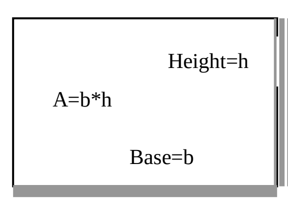
This activity allows students at every van Hiele level to experience success. When the students come together after working independently, group students of different van Hiele levels together to share strategies and solutions. This will allow students to learn from each other as well as to move from one level to the next (Malloy, 1999).

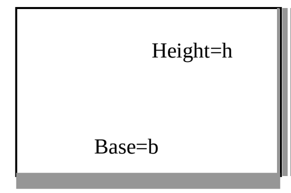
To make a connection to area with this activity, the teacher can have students work in cooperative groups to come up with answers to the following questions:

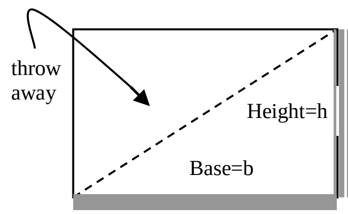
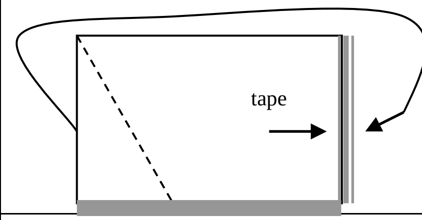
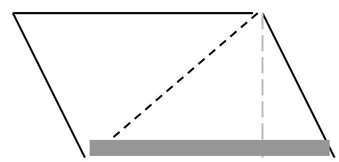
1. What is the area of the original figure?
2. What is the area of each new figure with a perimeter of 18?
3. Are all the answers in Question 2 the same? Why or why not?
4. Construct as many different rectangles as you can with an area of 24. Do they all have the same perimeter?
5. Construct as many different rectangles as you can with a perimeter of 16. Do they all have the same area?
6. What are some connections between area and perimeter?

(Malloy, 1999)

<u>Index Cards</u>
Once students have a conceptual understanding of the differences between area and perimeter, it may be necessary to provide an additional visual to support the differences in area formulas from one figure to the next (NCTM, 2000). The following illustrates how index cards may be used for this purpose.

1. Use an index card to represent a rectangle. Color the base edge gray and label it base. Color the height edge light gray and label it height. Write the formula: Area = base x height.	

<table>
<tr><th></th><th></th></tr>
<tr><td>2. Take a second index card and shade the sides the same color as in Step 1.</td><td>
</td></tr>
<tr><td>3. Now cut the triangle in half as shown. Throw away the piece with no color

Discuss the formula for the area of a triangle.
Area = (base x height) ÷ 2.</td><td>
</td></tr>
<tr><td>4. On a third card make a cut as shown and tape the two pieces together to form a parallelogram.</td><td>
</td></tr>
<tr><td>5. Cut the parallelogram in half and discuss the area of this triangle.</td><td></td></tr>
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Strategies to Teach Geometric Solids

In teaching geometric solids to middle school students, teachers and students will benefit from using the van Hiele phases of learning.

1. Inquiry/information
2. Directed Orientation
3. Explication
4. Free Orientation
5. Integration

The phases and related activities follow.

<u>Inquiry/information</u>
The purpose at this stage is to assess students' prior knowledge to guide the instruction of the unit of study (Crowley, 1987). To begin this phase, the teacher can ask the following types of questions:

- What is a cube?
- What is a cone?
- What is a prism?
- What is a sphere?

- How are they alike and how are they different?

Directed Orientation
During the directed orientation stage, the teacher can lead guided explorations of the solids. Activities may include the following:

- Gather different household items that resemble a cone, cube, sphere, cylinder, pyramid, and prism. Have students sort them by different teacher-described attributes.
- Have students trace each side of the solid to determine its net (a two-dimensional figure that when folded forms the surface of a three-dimensional object) (Boyd, Burrill, Cummins, Kanold, & Malloy, 1998). Students could then construct the object from the net.
- Using a flashlight, cast a shadow of each face of the solid onto the wall to help determine its net.
- Have the students use a Venn diagram and pictures of different solids to analyze similarities and differences between them.
- Create a line plot by looking at each solid and determining the number of edges, surfaces, or points for each.

Explication
In the third phase, students write and discuss the observations from the second phase. Students express their ideas about the solids (Crowley, 1987). Example activities include:

- Write a paragraph describing each of the solids; include diagrams with important parts labeled.
- Create a graphic organizer showing the relationship between the different types of solids.
- Discuss the similarities and differences of the solids.

Free Orientation
At the next phase, free orientation, students learn about the solids by performing more complex tasks. These tasks will be multi-step and will require a higher level of thinking than in directed orientation. Examples include:

- Fill solids with rice, sand, or water and explore properties of volume.
- Cut apart cardboard solids such as cereal boxes to explore the surface area of solids.

Integration
In the final phase, students do not learn any new material. Instead they review and summarize the work done in this unit through activities such as the following:

- Create a poster describing the properties of the solids.
- Demonstrate understanding of the solids through a brochure developed to teach their peers (Crowley, 1987).

Strategies to Teach Transformations

The following types of transformations should be developed at the middle school level:
- Translations--a transformation that glides all points of a figure the same distance in the same direction.

- Rotations--a transformation that rotates all points of a figure about a point through an angle of x degrees.
- Reflections-- a transformation in which a line of reflection acts like a mirror reflecting points to their image.
- Dilations-- a transformation that expands or contracts a figure by *k* times, where *k* is the scale factor (Jurgensen, Brown, & Jurgensen, 1992).

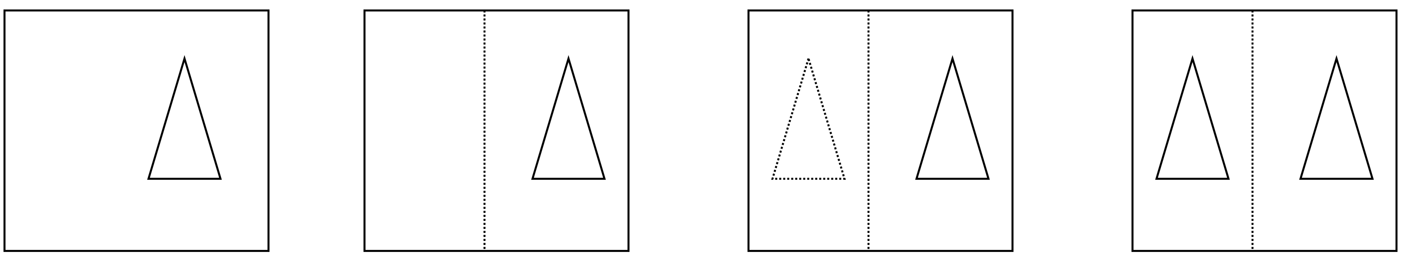
Middle school teachers can use one of the following three processes to teach transformations dependent on the students' readiness and van Hiele level.

1. Tracing paper
2. Coordinate geometry
3. Dynamic geometry software

Tracing Paper

Students at the Visualization level are focusing on properties of the whole figure and can begin their exploration of transformations using tracing paper. For example, students can draw a figure and then use the paper to do the transformation as illustrated below.

Step 1 **Step 2** **Step 3** **Step 4**

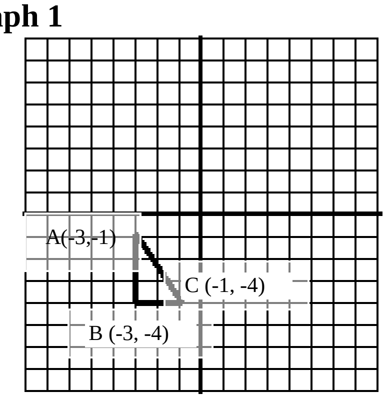


Draw a figure on tracing paper. **Fold a line of reflection on the paper** **Trace the image with paper folded** **Open the paper to reveal both images.**

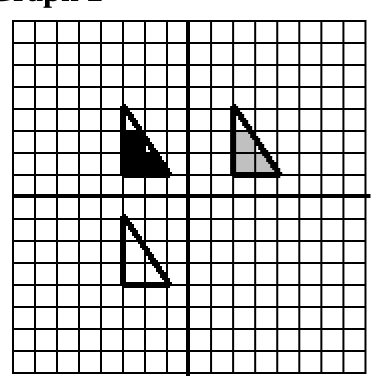
Coordinate Geometry

To differentiate, teachers can use coordinate geometry to investigate the properties of transformations with students who demonstrate the ability to work with more than one concept at a time. A full understanding of the coordinate system is required for this type of exploration. For example:

Graph 1



Graph 2



Students can be given a figure as in Graph 1. Students are then asked to translate the figure up 5 units and then to the right 5 units. Students must redraw the figure and name the new coordinates of the figure's vertices. In the above example the black triangle's vertices will be A (-3, 4), B (-3, 1) and C (-1, 1) and the vertices of the gray triangle will be A (2, 4), B (2, 1) and C (4, 1).

Dynamic Geometry Software

Finally, students who are at the informal deductive level and are able to make connections between different types of figures and their properties can use a dynamic geometry software program to investigate the properties of transformations. For example, students can do the following:

- Investigate reflections using different types of parallelograms to determine the properties of reflections.
- Rotate a point, line, and a figure looking at the similarities and differences among the three.
- Translate a figure using software to describe the outcome.
- Dilate different types of figures using both whole numbers and numbers between zero and one to develop a conjecture.
- Describe what happens when a combination of transformations occur.

Conclusion

Middle school mathematics teachers have an important role to play in the development of concepts that students will need as they take a high school geometry course. Therefore, it is essential that teachers determine their students' geometric understanding through the use of van Hiele levels, and then use research-based strategies to move them to the next level. Activities may be organized for differentiation to provide each student the material and time necessary to fully grasp the concepts. By understanding where students are in their geometric comprehension, teachers can best meet their needs, and will be more successful in teaching the middle school geometry curriculum.

References

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Crowley, M.L. (1987). The van Hiele model of the development of geometric thought. In M. Montgomery Lindquist (Ed.), *Learning and teaching Geometry, K-12, 1987 Yearbook of the National Council of the Teachers of Mathematics* (pp.1-16). Reston, VA: National Council of Teachers of Mathematics.

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Additional Resources

Resources are available for loan through the T/TAC W&M library. Visit the website http://education.wm.edu/centers/ttac/index.php, for a complete listing of all materials. Select the library link off the home page and enter "Mathematics" as the subject of the search.

Other Suggested Resources

Title	Author
Geometry: An Investigative Approach	O'Daffer, P.G. & Clemens, S.R.
Geometry Labs	Picciotto, H.

This *Considerations Packet* was prepared by Elizabeth M. O'Brien, October 2004.