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Algebra Strategies for Middle School Considerations Packet

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Algebra Strategies for Middle School

This *Considerations Packet* addresses strategies that middle school teachers can implement in the teaching of algebraic thinking. Algebra is a strand found in each of the middle school grade levels in the Virginia Standards of Learning. It is also one of the areas addressed by the National Council of Teachers of Mathematics (NCTM) in *Principles and Standards for School Mathematics*. It is of utmost importance that middle school teachers address these standards and work to develop algebraic thinking in their students. Algebra has been called the gatekeeper course for all of mathematics and many career choices. If we are to expect success in high school mathematical courses, then it is necessary to encourage student success with algebra at an early age.

The Algebra strategies included in this packet address the following topics: exploring patterns, graphs, symbolic manipulation, technology as an aid for understanding, discourse in the algebra classroom, and writing about algebraic thinking. Additional references and resources are provided at the end of the packet.

Exploring Patterns

At the middle school level, students are expected to represent, analyze, and make generalizations about patterns. These patterns should be linear in nature, arising from a constant rate of change. In each pattern, the students should be able to use multiplication and addition to find the relationship between the two sets of numbers. Students should look at patterns through the use of tables, graphs, and symbolic representation (NCTM, 2000). Students who study and represent patterns in both numerical and geometric formats begin to develop a solid algebraic understanding of functions (NCTM, 2000).

<u>Hundreds Charts</u> (Thompson & Mayfield-Ingram, 1998)
Exploration of patterns can begin by using a hundreds chart which is familiar to most students.

1. Have students highlight multiples of 2, 3 or 4 on the chart. Some students may still call this "skip counting."
2. Create a table with one column labeled n: nth and the other column labeled v: value. For example:

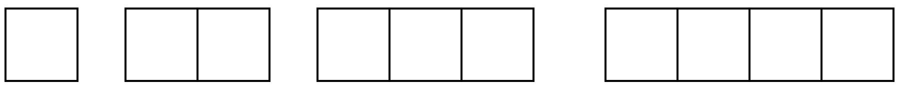
n: nth	v: value
1^{st}	3
2^{nd}	6
3^{rd}	9
4^{th}	12
5^{th}	15

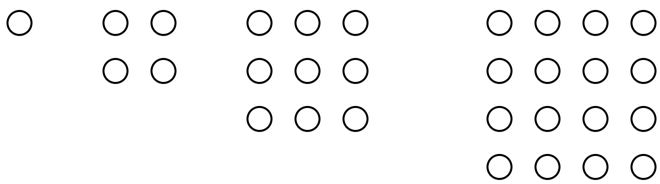
3. Have students study the chart to help them understand the relationship between the term and the value. Questions should include: How can I find the 34^{th} term? When the value is 120, what term am I on? How can I find any value given a term? Can I represent this as an equation?
4. To extend the activity, have students think of the numbers in the table as coordinate pairs. Before graphing them, have students predict what the graph will look like.
5. Ask students if the points on the graph are connected with a line, do the points between the graphed points fit the pattern?

6. Have students make predictions about what the table and its graph will look like when a different number is used. Have students then test their predictions.

<u>Geometric Patterns</u> (Thompson & Mayfield-Ingram, 1998)
Students need to see the connection between the different areas of mathematics. Mathematics becomes more meaningful when they can see the relationships between these different areas of the field. The use of geometric patterns to understand geometry provides an opportunity for students to explore patterns.

1. Present students with geometric patterns, for example:

a. 

b. 

2. Have students extend the pattern to show at least the next two steps in the pattern.
3. Use a table to show each step in the pattern. Have the left hand side of the table be the step and the right hand side be the number of units, sides, or circles in the pattern.
4. Ask students to talk about the patterns and how they would describe them in words.
5. Use a coordinate plane to graph the pattern, thinking of each pair of numbers in the table as a coordinate pair.
6. Discuss what the graph looks like and why.

Graphs

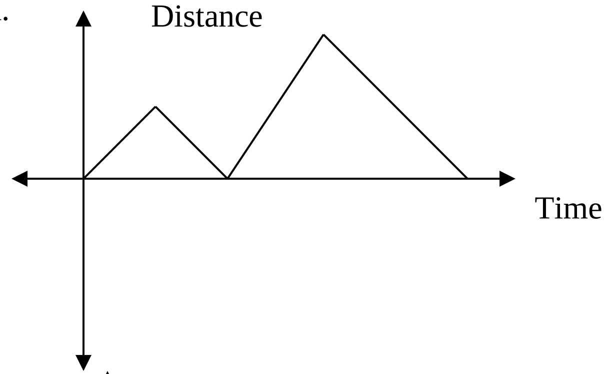
In their study of algebra, middle school students are expected to analyze graphs with linear relationships and discuss the nature of the change in quantities (NCTM, 2000). Because graphs are encountered throughout all of mathematics as well as in many other disciplines, this is an area that requires focused attention.

<u>Graphs to Equations</u> (Thompson & Mayfield-Ingram, 1998)
Teachers often emphasize graphs after students have solved equations or have looked at functions and patterns. It is important that graphs be included along with the algebraic and numeric manipulations. One strategy is to begin with a graph and work backwards to a table and algebraic rule.

1. Present students with a linear graph.
2. Have students create a table that contains points on the graph that are sequential. (This will make finding the pattern easier.)
3. Using the table, facilitate a discussion of patterns that occur in the table.
4. Write the pattern in words.
5. Use symbols to represent the words.

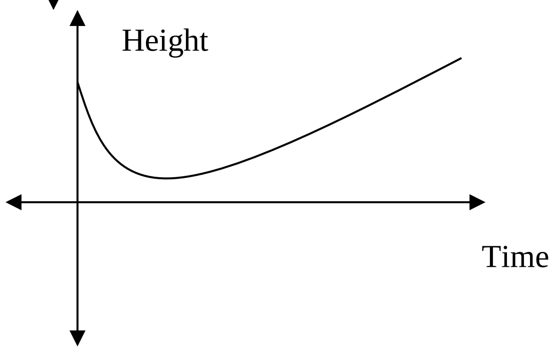
<u>Interpreting Graphs</u> (Burrill & Hopfensperger, 1998)
The more opportunities that students are given to verbalize interpretations from a graph, the more successful they will become at the process. Interpreting graphs is an essential skill in the study of algebra.
Examples of graphs and questions that can be asked are presented below. (Solutions at end of packet.)

a.



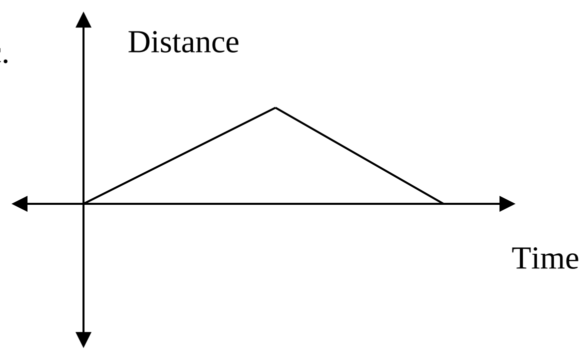
This graph represents the distance a person is from their house as they take a walk.
1. As time passes, what happens to the person's distance from home?
2. When the graph intersects the x-axis, what does this represent?

b.



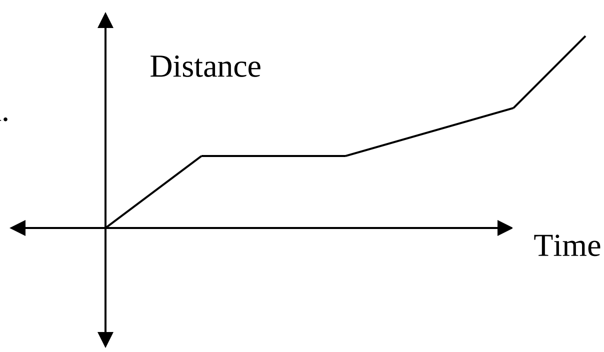
This graph represents the movement of a model airplane.
1. Describe the distance at time = 0.
2. As time passes, how would you describe the height of the plane?

c.



This graph represents a girl leaving her house and taking her dog for a walk.
1. What does the highest point on the graph represent?
2. How would you describe the walk on which the girl takes her dog?

d.



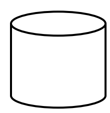
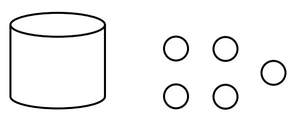
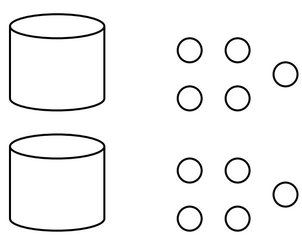
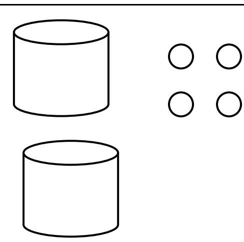
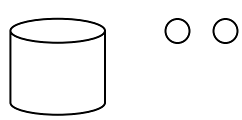
This graph represents the movement of a school bus. Write a paragraph describing what you think is happening to the bus during the time shown on this graph.

Symbolic Manipulation

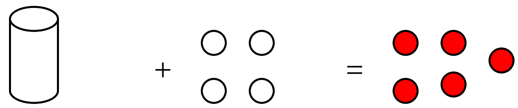
One of the most challenging aspects of algebra for many students is understanding the abstract nature of solving equations and manipulating symbols. In order to help students succeed, it is necessary to begin their study of solving equations with the use of a manipulative to concretely represent the symbolic equations (Driscoll, 1999).

There are many commercially available products that are valuable for demonstrating to students the concepts encountered when solving equations. Some that are widely available include: Algebra lab gear, hands-on equations, and algebra tiles. While these all do an excellent job, it is possible to create your own. Using beans and small cups, we can illustrate many of the same concepts as the commercial products.

<u>Math Tricks</u> (Thompson & Mayfield-Ingram, 1998)
By using "math tricks" students' interest will be piqued and elements of symbolic manipulation can be taught. One of these sequences is shown below.

Think of a number.		n
Add 5 to your number		n + 5
Double your number (or multiply by 2)		n + 5 + n + 5 or 2(n + 5) or 2n + 10
Subtract 6		2n + 4
Divide by 2		n + 2
Tell me your new number and I will tell you what number you started with.		

Manipulatives for solving equations (Thompson & Mayfield-Ingram, 1998)
When solving the one-step equation, $x + 4 = -5$, the teacher can represent the equation as follows:

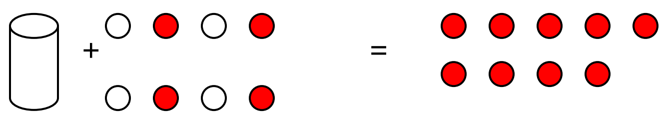


Using a small plastic cup or a film canister to represent the unknown quantity and beans painted red on one side to represent known quantities enables students to represent the equation. The teacher explains that students are looking for the unknown quantity of beans that will fill the cup to make both sides equal or balanced.

Concepts that need to be taught and reinforced through this method include:

1. Having different colors stand for positive and negative numbers.
2. How to represent a variable.
3. Forming a zero by having equal quantities of positive and negative beans, emphasizing the idea that they will "cancel each other out."

To continue this example, the teacher places 4 red beans on the left side of the equation to "cancel out" the 4 white beans. Then she places 4 red beans on the right hand side of the equation to keep the equation balanced.



Students should spend considerable time in class working with concrete manipulatives to problem solve. After students are comfortable with the process of representing variables and creating a zero, they can move to drawing pictures to represent the equations. Finally, after much practice, students can move to the abstract manipulation of numbers and symbols.

As students move to the symbolic stage, many will benefit from using a different colored pencil to show the step where a zero is created and both sides of the equation are kept balanced. (The example below shows the use of bold numbers to indicate where color could be used.)

$$\begin{array}{rcl} x + 4 & = & -5 \\ \underline{\mathbf{-4}} & & \underline{\mathbf{-4}} \\ x & = & -9 \end{array}$$

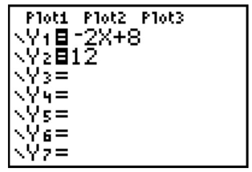
Technology as an Aid for Understanding

Using scientific and graphing calculators can greatly enhance students' understanding of algebraic concepts. The graphing calculators enable students to visualize the abstract concepts of algebra. Technology should be used to aid and enhance the learning and never to bypass the teaching of a concept. Without basing the calculator skill in a meaningful context, the skill will not be retained by the student (Posamentier, Hartman, & Kaiser, 1998).

If students begin their study of algebra with understanding functions and examining tables and graphs, then there will be a logical step toward solving equations through the use of tables and graphs. At the middle school level, students are dealing primarily with linear equations and solving one- and two-step equations. The graphing calculator can be an effective tool to enhance their understanding because it allows them to visualize many of the abstract concepts in the curriculum.

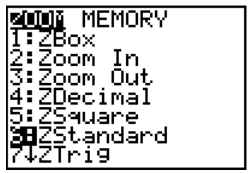
<u>Bridging equation solving and graphing with the graphing calculator</u> (Yerushalmy & Gilead, 1999)
If students understand how to graph a linear equation with the use of a table, or the slope and a point, then looking at solving an equation through graphs will be a natural progression. (The following examples are shown with the use of a Texas Instrument graphing calculator, but a comparable Casio graphing calculator could also be used.)

For example: Solve $-2x + 8 = 12$

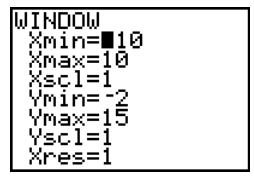


1. Go to y= on the graphing calculator and enter as shown.

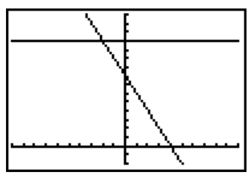
2. Zoom 6: Standard and enter.



3. Notice only one line can be seen. Encourage students to think about the viewing window that is needed in order to see both graphs. One possibility is shown. Note: Students can also be shown how to Zoom 3: Zoom Out



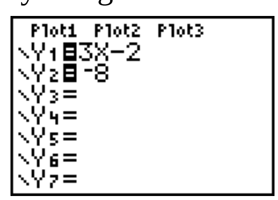
4. Now examine the graph again.



5. At this point, have students discuss how to find the solution using the graph. There are several possibilities, including tracing and checking x values, using the "calculate" function key, or using the x= and plugging in values.

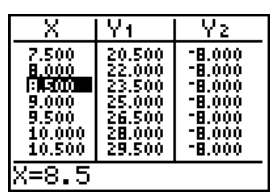
<u>Using tables on the graphing calculator</u> (Yerushalmy & Gilead, 1999)
The above process can be repeated, but the emphasis placed on finding a solution by using a table instead of the graph.

For example: Solve $3x - 2 = -8$



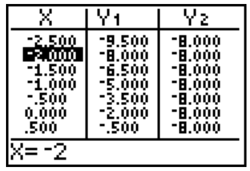
1. Again, have the students begin by going to y = and putting the equations into y1 and y2.

2. To examine the table of values, go to 2nd graph.



3. Have students discuss where they can look on the table to find their solution. Show them how to use 2nd window to aid in setting up the table so that they can look at particular values of x in which they are most interested.

4. Now the solution can be found.



Discourse in the Algebra Classroom (Sherin, Mendez, & Louis, 2000)

Middle school students are expected to communicate their understanding of algebraic concepts to their teachers, their peers, and their parents (NCTM, 2000). To facilitate this process, teachers must have methods for instructing students that foster a community where discourse is valued and encouraged. When discourse is present in the classroom, teachers benefit by having a clearer picture of students' understanding and instructional level. Because mathematical discourse influences student learning, it is essential that it be encouraged and employed (Sherin, Mendez, & Louis, 2000).

The following three step strategy, when modeled and taught, has been found to strengthen student discussions in classrooms where discourse becomes a part of teaching and learning and where everyone participates.

Step 1: Explain

The goal of this step is to have students justify their statements. Questions for teachers to ask and ways to model this include:

a. "Explain why you think that is the answer."
b. "Show the class your thinking."
c. "Elaborate on your answer and give more details."
d. "Discuss different methods for solving the same problem."
e. Praise and encourage students who provide explanations without prompting.
f. Establish early on in the year that explanations are part of answers.

Step 2: Build

At this step, students are engaged in the discussion and learning process because they are being taught to listen to each other's ideas and contribute to what their peers have said. To encourage this, teachers should ask:

a. What do people think about (child's name) explanation?
b. Rephrase and compare two students' thinking and ask for opinions.
c. Can anyone provide more details to (child's name) explanation?
d. Rephrase a student's thinking and ask for additional comments and feedback.

Step 3: Go Beyond

In the "go beyond" step, students are required to make generalizations about their thinking. Students offer an answer to the problem, and then

a. Model the technique for students by synthesizing the content presented.
b. Provide problems that are rich in mathematics and require higher-level thinking.

c. Encourage conjectures and the testing of them at this step.

Writing about Algebraic Thinking

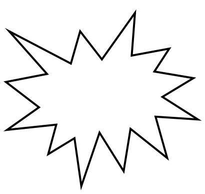
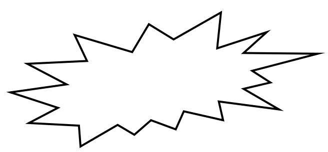
Research supports that teachers who include impromptu writing prompts in their mathematics classes have an enhanced understanding of students' algebraic thinking. Teachers' instructional practices are influenced as well (Miller, 1992).

Three types of writing prompts include: content prompts, process prompts, and attitudinal prompts.

<u>Content Prompts</u>
Content prompts involve open-ended problems, many of which can be solved in more than one way. It is essential that students explain their thinking and justify their understanding.

For example:

1. Last night Joseph's dog took two bites out of his homework. Can you determine what goes in the holes?

12 +  +  = 57

2. Describe this pattern in at least two different ways.

10, 5, 0, -5, -10….

<u>Process Prompts</u>
Process prompts allow teachers to get a deeper understanding of their students' thinking. These prompts are not just about doing problems, but understanding the *how* and *why* of what students do.

For example:

1. Jessie has forgotten how to solve the equation $3x + 2 = 8$. Explain how she can find the answer two different ways.

2. Your best friend was sick today and not in school. Write down how you will explain the process of creating a table for the function $2x - 8$. Provide an example as well.

3. Mia is confused; she always thought that when you multiplied, you would get a bigger number like $3 \times 5 = 15$. But she just put 0.3×0.5 in her calculator and she got 0.15. Why is she getting a smaller number?

Attitudinal Prompts
Attitudinal prompts give mathematics teachers a window into their students' emotions about algebra. Teachers can learn many things about their students that they would not learn from traditional pencil-and- paper problems by using these types of prompts.

For example:

1. Describe one thing that helps you to learn in this classroom. Describe one thing that keeps you from learning.

2. Describe how it makes you feel when you have to work a problem at the board.

3. If you could be any number, what would it be and why?

4. Describe what you learned in this unit. Do you feel that you have a good understanding of the concepts studied? If not, what has kept you from learning them?

Conclusion

Algebra is the gatekeeper for future math courses. In order for students to be successful in algebra, it is essential that the middle school curriculum provide students with opportunities to develop their algebraic thinking. The strategies presented in this *Considerations Packet* provide teachers with research-based ideas that will promote algebraic understanding for all students. Incorporating these concepts will provide students with the opportunity to experience success in middle school mathematics and in algebra.

Solutions to Interpreting Graphs

a. 1. When the graph increases and the slope of the line is positive, the distance from the house increases. When the graph decreases and the slope of the line is negative, the distance from the house decreases. As time passes, the graph represents the person moving away from the house and then towards it, away from the house and then towards it.
2. When the graph touches the x- axis the distance from the person to the house is zero. Therefore, the person must be at the house when the graph intersects the x-axis.

b. 1. At time equal to zero the plane is above the ground.
2. As time passes, the plane's height begins to decrease and then the height increases again.

c. 1. The highest point on the graph represents the point in time when the girl is farthest from the house.
2. I would describe the path the girl takes as "out and back". The girl's distance from home steadily increases and then steadily decreases.

d. One possible description of the bus' movement would be that it leaves a school and then comes to a stop where it stays still for a few minutes while the children depart. Then the distance increases again as it moves down the street. It increases more sharply (change in slope) when the speed of the bus increases and the distance continues to increase.

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This *Considerations Packet* was prepared by Elizabeth M. O'Brien, August 2004.