| **3.20 The student will**  a) investigate the identity and the commutative properties for addition and multiplication; and  b) identify examples of the identity and commutative properties for addition and multiplication. | | |
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| UNDERSTANDING THE STANDARD(Background Information for Instructor Use Only) | ESSENTIAL UNDERSTANDINGS | ESSENTIAL KNOWLEDGE AND SKILLS |
| * Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic relationships. These relationships remain true regardless of the numbers. * The commutative property for addition states that changing the order of the addends does not affect the sum (e.g., 4 + 3 = 3 + 4). Similarly, the commutative property for multiplication states that changing the order of the factors does not affect the product (e.g., 2 × 3 = 3 × 2). * The identity property for addition states that if zero is added to a given number, the sum is the same as the given number. The identity property of multiplication states that if a given number is multiplied by one, the product is the same as the given number. * A number sentence is an equation with numbers (e.g., 6 + 3 = 9; or 6 + 3 = 4 + 5). | All students should   * Understand that mathematical relationships can be expressed using number sentences. * Understand the identity property for addition. * Understand the identity property for multiplication. * Understand the commutative property of addition. * Understand the commutative property of multiplication. * Understand that quantities on both sides of an equal~~s~~ sign must be equal. * Understand that quantities on both sides of the not equal sign are not equal. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to   * Investigate the identity property for addition and determine that when the number zero is added to another number or another number is added to the number zero, that number remains unchanged. Examples of the identity property for addition are 0 + 2 = 2; 5 + 0 = 5. * Investigate the identity property for multiplication and determine that when the number one is multiplied by another number or another number is multiplied by the number one, that number remains unchanged. Examples of the identity property for multiplication are 1 x 3 = 3; 6 x 1 = 6. * Recognize that the commutative property for addition is an order property. Changing the order of the addends does not change the sum (5 + 4 = 9 and 4 + 5 = 9). * Recognize that the commutative property for multiplication is an order property. Changing the order of the factors does not change the product (2 × 3 = 3 × 2). * Write number sentences to represent equivalent mathematical relationships (e.g., 4 x 3 = 14 - 2). * Identify examples of the identity and commutative properties for addition and multiplication. |

| **4.16 The student will**  **a) recognize and demonstrate the meaning of equality in an equation; and**  **b) investigate and describe the associative property for addition and multiplication.** | | |
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| **UNDERSTANDING THE STANDARD**  **(Background Information for Instructor Use Only)** | **ESSENTIAL UNDERSTANDINGS** | **ESSENTIAL KNOWLEDGE AND SKILLS** |
| Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic relationships. These relationships remain true regardless of the numbers.  The commutative property for addition states that changing the order of the addends does not affect the sum (e.g., 4 + 3 = 3 + 4). Similarly, the commutative property for multiplication states that changing the order of the factors does not affect the product (e.g., 2 × 3 = 3 × 2).  The associative property for addition states that the sum stays the same when the grouping of addends is changed [e.g., 15 + (35 + 16) = (15 + 35) + 16]. The associative property for multiplication states that the product stays the same when the grouping of factors is changed [e.g., 6 × (3 × 5) = (6 × 3) × 5]. | **All students should**  Understand that mathematical relationships can be expressed using equations.  Understand that quantities on both sides of an equation must be equal.  Understand that the associative property for addition means you can change the groupings of three or more addends without changing the sum.  Understand that the associative property for multiplication means you can change the groupings of three or more factors without changing the product**.** | **The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**  Recognize and demonstrate that the equals sign (=) relates equivalent quantities in an equation.  Write an equation to represent equivalent mathematical relationships (e.g., 4 × 3 = 2 × 6).  Recognize and demonstrate appropriate use of the equals sign in an equation.  Investigate and describe the associative property for addition as (6 + 2) + 3= 6 + (2 + 3).  Investigate and describe the associative property for multiplication as (3 x 2) x 4 = 3 x (2 x 4). |

**5.19 The student will investigate and recognize the distributive property of multiplication over addition.**

| **UNDERSTANDING THE STANDARD**  **(Background Information for Instructor Use Only)** | **ESSENTIAL UNDERSTANDINGS** | **ESSENTIAL KNOWLEDGE AND SKILLS** |
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| * The distributive property states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products (e.g.,            3(4 + 5) = 3 x 4 + 3 x 5,  5 x (3 + 7) = (5 x 3) + (5 x 7); or           (2 x 3) + (2 x 5) = 2 x (3 + 5).   * The distributive property can be used to simplify expressions (e.g., 9 x 23 = 9(20+3) =180+ 27 = 207; or 5 x 19 = 5(10 + 9) = 50 + 45 = 95). | **All students should**  Understand that the distributive property states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products.  Understand that using the distributive property with whole numbers helps with understanding mathematical relationships.  Understand when and why the distributive property is used. | **The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**  Investigate and recognize the distributive property of whole numbers, limited to multiplication over addition using diagrams and manipulatives.  Investigate and recognize an equation that represents the distributive property, when given several whole number equations, limited to multiplication over addition. |

| **6.19 The student will investigate and recognize**  **a) the identity properties for addition and multiplication;**  **b) the multiplicative property of zero; and**  **c) the inverse property for multiplication.** | | |
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| **UNDERSTANDING THE STANDARD**  **(Background Information for Instructor Use Only)** | **ESSENTIAL UNDERSTANDINGS** | **ESSENTIAL KNOWLEDGE AND SKILLS** |
| Identity elements are numbers that combine with other numbers without changing the other numbers. The additive identity is zero (0). The multiplicative identity is one (1). There are no identity elements for subtraction and division.  The additive identity property states that the sum of any real number and zero is equal to the given real number (e.g., 5 + 0 = 5).  The multiplicative identity property states that the product of any real number and one is equal to the given real number (e.g., 8 · 1 = 8).  Inverses are numbers that combine with other numbers and result in identity elements.  The multiplicative inverse property states that the product of a number and its multiplicative inverse (or reciprocal) always equals one (e.g., 4 · = 1).  Zero has no multiplicative inverse.  The multiplicative property of zero states that the product of any real number and zero is zero.  Division by zero is not a possible arithmetic operation. Division by zero is undefined. | How are the identity properties for multiplication and addition the same? Different? For each operation the identity elements are numbers that combine with other numbers without changing the value of the other numbers. The additive identity is zero (0). The multiplicative identity is one (1).  What is the result of multiplying any real number by zero?  The product is always zero.   * Do all real numbers have a multiplicative inverse? No. Zero has no multiplicative inverse because there is no real number that can be multiplied by zero resulting in a product of one. | **The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**  Identify a real number equation that represents each property of operations with real numbers, when given several real number equations.  Test the validity of properties by using examples of the properties of operations on real numbers.  Identify the property of operations with real numbers that is illustrated by a real number equation.  NOTE: The commutative, associative and distributive properties are taught in previous grades. |

| **7.16 The student will apply the following properties of operations with real numbers:**  **a) the commutative and associative properties for addition and multiplication;**  **b) the distributive property;**  **c) the additive and multiplicative identity properties;**  **d) the additive and multiplicative inverse properties; and**  **e) the multiplicative property of zero.** | | |
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| **UNDERSTANDING THE STANDARD**  **(Background Information for Instructor Use Only)** | **ESSENTIAL UNDERSTANDINGS** | **ESSENTIAL KNOWLEDGE AND SKILLS** |
| * The commutative property for addition states that changing the order of the addends does not change the sum (e.g., 5 + 4 = 4 + 5).   The commutative property for multiplication states that changing the order of the factors does not change the product (e.g., 5 · 4 = 4 · 5).  The associative property of addition states that regrouping the addends does not change the sum [e.g., 5 + (4 + 3) = (5 + 4) + 3].  The associative property of multiplication states that regrouping the factors does not change the product [e.g., 5 · (4 · 3) = (5 · 4) · 3].  Subtraction and division are neither commutative nor associative.  The distributive property states that the product of a number and the sum (or difference) of two other numbers equals the sum (or difference) of the products of the number and each other number [e.g., 5 · (3 + 7) = (5 · 3) + (5 · 7), or 5 · (3 – 7) = (5 · 3) – (5 · 7)].  Identity elements are numbers that combine with other numbers without changing the other numbers. The additive identity is zero (0). The multiplicative identity is one (1). There are no identity elements for subtraction and division.  The additive identity property states that the sum of any real number and zero is equal to the given real number (e.g., 5 + 0 = 5).  The multiplicative identity property states that the product of any real number and one is equal to the given real number (e.g., 8 · 1 = 8).  Inverses are numbers that combine with other numbers and result in identity elements [e.g., 5 + (–5) = 0; · 5 = 1].  The additive inverse property states that the sum of a number and its additive inverse always equals zero [e.g., 5 + (–5) = 0].  The multiplicative inverse property states that the product of a number and its multiplicative inverse (or reciprocal) always equals one (e.g., 4 · = 1).  Zero has no multiplicative inverse.  The multiplicative property of zero states that the product of any real number and zero is zero.  Division by zero is not a possible arithmetic operation. Division by zero is undefined. | * Why is it important to apply properties of operations when simplifying expressions?  Using the properties of operations with real numbers helps with understanding mathematical relationships. | **The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**   * Identify properties of operations used in simplifying expressions. * Apply the properties of operations to simplify expressions. |