Hands-on Problem Solving

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What is problem solving?

- The process of applying previously acquired knowledge to new and unfamiliar situations.
- If you know how to solve the problem immediately, it is not really a problem.

Is this a good problem?

- Sam has 3 cookies. He eats 2 of them. How many does he have left?
- How many diagonals does a pentagon have?

NCTM promotes

- Teaching *for* problem solving - Teaching skills, then providing problems to practice those skills
- Teaching *about* problem solving - Teaching the process
- Teaching *through* problem solving - Teaching content through real context, problems, situations, and models

4-step Problem Solving Process (teaching *about* problem solving)

- Understand the problem (Restate in your own words, draw a diagram of the situation, ...)
- Make a plan (Decide on a strategy)
- Carry out the plan (Show all your thinking.)
- Look back (Did you answer the question that was asked? Does your answer make sense? Is there another way to do the problem?....)

Heuristics (strategies)

- Draw a picture or diagram
- Look for a pattern
- Guess and check
- Make a table or a chart
- Try a simpler form of the problem
- Make an organized list
- Write and equation
- Act it out/set up a simulation

Jack and Jill

Jack and Jill have $20 when they put their money together. Jill has $4 more than Jack. How much does Jack have?

Good Math Teaching ...

- Starts with concrete materials.
- Moves to semi-concrete (pictorial) representation.
- Ends with abstract

How many outfits can be put together...

- If you have two different-colored shirts and two different-colored shorts?
- If you have three different-colored shirts and two different-colored shorts?
- ...three colors of shirts and four colors of shorts?

What makes a good problem?

- Engages students in thinking about and developing mathematics they need to learn
- Begins where the students are, with multiple entry and exit points
- Is problematic or engaging due the mathematics, not the context (candy???)
- Demands higher-level thinking for making connections, analyzing information, and justifying to draw conclusions.

Let's Make a Deal (The Monty Hall Problem)

- Three doors. One has a huge prize behind it. The other two doors have goats behind them.
- You choose door A. Then the host opens door B and shows you it has a goat behind it.
- Should you stay with door A or switch to door C?

Debrief your problems!

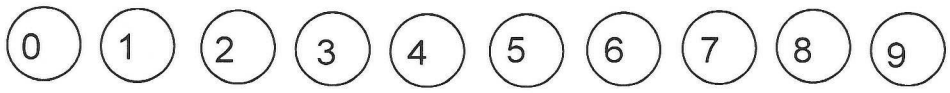
- Have students share their methods.
- Value attempts that do not result in a perfect solution.
- Allow for some long term problems. Present a problem on Monday. Do the "Understand the problem" part together. Have students think about it for a few days and then work on it a few days later.
- Keep a problem solving journal.

Number Bracelets

http://www.geom.uiuc.edu/~addingto/number_bracelets/number_bracelets.html

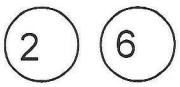
The origin of this game, in this form, is an activity by Marilyn Burns (*About Teaching Mathematics, a K-8 Resource*, Math Solutions Publications, 1992.) Another version can be found in Joan Cotter's book *Math Card Games*.

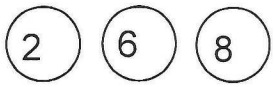
Imagine that you have lots of beads, numbered from 0 through 9, as many as you want of each kind.

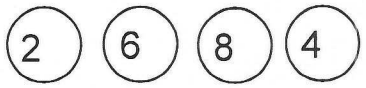


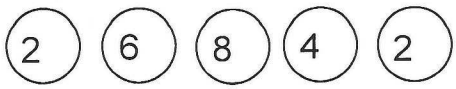
Here are the rules for making a number bracelet:

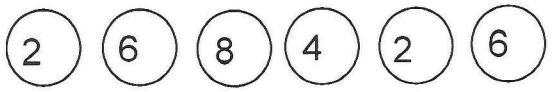
- Pick a first and a second bead. They can have the same number. Imagine putting them on a string.
- To get the third bead, add the numbers on the first and second beads. If the sum is more than 9, just use the last (ones) digit of the sum.
- To get the next bead, add the numbers on the last two beads you used, and use only the ones digit. So to get the fourth bead, add the numbers on the second and third beads, and use the ones digit. Keep going until you get back to the first and second beads, in that order.
- Drop the beads that start the repeating pattern. Tie your string off and you have a "bracelet."

Example: Choose 2 and 6 for the first and second beads: 

The third bead is 2 + 6 = 8 : 

To get the fourth bead, add 6 and 8, then use only the ones digit: 6 + 8 = 14; use 4 

Next bead: 8 + 4 = 12; use 2: 

Then 4 + 2 = 6: 

Since the last two beads start a repeating pattern, drop them, leaving only 2, 6, 8, and 4. Tie your string and you have a completed bracelet of length 4 in this case.

Questions: (Answers can be found at the website at the top of this page.)

- How long (or short) a number bracelet can you make?
- Will a number bracelet always loop back to the beginning, or can you have a string of beads that never repeats?
- How many different starting pairs of beads are there?
- How many different number bracelets are there?
- If you start with the same two beads, but in the opposite order, do you get the same bracelet? Do you get the same bracelet in reverse?

Nine Digits Problem

Use the digits 1-9 to make a correct sum of the form given below. You may tear off the strip at the bottom of the page, tear off the digits, and use them as a manipulative

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  X X X
+ X X X
  X X X
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Some questions to ask:

Ask for a solution from a volunteer. (Ex: 419+238 = 657) Write it on the board. Ask: How could you use this solution to find another solution?

Take your sample solution above and interchange the hundreds digits (i.e., the 4 and the 2). Is the sum different from the first solution? Explain.

How many solutions do can you find that have the sum 657?

Can you interchange any two columns to find another solution? Does this always work? Explain.

Is there any position that the digit 9 cannot be placed?

Will all solutions require regrouping ("carrying")?

What is the smallest sum that can be made? The largest sum? How do you know?

How many solutions do you think there are? How could we find out?

How can we make a cataloging system to file our list of solutions?

What patterns do you notice in the list of solutions to the problem?

1 2 3 4 5 6 7 8 9

Some problems for elementary grades:

1) There are 14 cookies on plate A. There are 4 more cookies on plate B than on plate A. There are 2 fewer cookies on plate C than on plate B. How many cookies are there on plate C?

2) There are 4 friends playing together: There are two boys and two girls. Each boy gives a high five to each girl. How many high fives are exchanged?

3) There are 3 students playing soccer. Their teacher wants to choose 2 of them to be team leaders. How many different pairs of leaders can their teacher choose?

4) Olivia has 3 t-shirts and 2 pairs of jeans. She has to match each t-shirt with a pair of jeans. How many different ways can she match them?

5) Dad has four boxes: 1, 2, 3, and 4. Box 1 is heavier than box 2 but lighter than box 3. Box 2 is lighter than box 3 but heavier than box 4. Which box is the heaviest?

6) There are 4 students at a table. Abigail is younger than Landon but older than Alexandra. Lucas is older than Alexandra but younger than Abigail. Who is the youngest in the group?

7) Mia has 20 marbles. Her brother, Anthony, has 5 fewer marbles than Mia. How many marbles does Anthony have?

8) A group of 5 students need to shake hands with one another. How many handshakes are exchanged altogether?

9) Mrs. Williams is making sandwiches. She has two types of bread: white bread and wheat bread. She has three types of spread: turkey, ham, and peanut butter. How many types of sandwiches can she make?

10) Fiona is in line to buy a milkshake. She is the sixth person from the front and the second person from the end of the line. How many people are in the line?

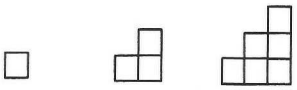
11) Mr. Martinez and Mr. Jackson travel from City A to City B. They leave city A at 1:00 pm. Mr. Martinez arrives at city B at 5:00 pm. Mr. Jackson arrives at city B two hours later. How many hours does Mr. Jackson take to travel from city A to city B?

12) Joshua has 4 coins: a 50-cent coin, a 25-cent coin, a 10-cent coin, and a 5-cent coin. How many different combinations can he make with any 2 coins.

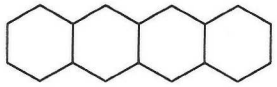
Some Middle Grades Problems

1. Find all the different ways that you can make a train of length 2 with Cuisenaire® rods. How many trains of length 3? Length 4? etc. (a different order counts as a separate way)

2. Count the squares and continue the pattern. How many squares will be in the 10^{th} drawing? The 100^{th} drawing?



3. Using pattern blocks, make a line of hexagons that share a common side. Find the perimeter if there is only one hexagon; 2 hexagons; 3 hexagons, etc.



4. Make a correct sum using the digits 1 through 9 only once each.

5. Make change for a dollar in all the ways you can.

6. A ribbon is 56 inches long. It is cut into two pieces. One piece is 1/7 as long as the other piece. How long is each piece?

7. Patti has $12 more than Shala and $15 more than Liz. Together all three have $87. How much money does Patti have?

8. If it takes a team of 4 people 15 days to do half of the job, how long will it take a team of ten people to finish the remaining half of the job (assuming each person on both teams works at the same rate)?

9. Eight adults and two children need to cross a river. A small boat is available that can hold 1 adult or 1 or 2 children (i.e., 3 possibilities: 1 adult in a boat, 1 child in a boat, or 2 children in a boat). Everyone can row the boat. How many one-way trips does it take for all of them to cross the river?

10. See this sample from Math Olympiads for the Elementary and Middle School:
http://www.moems.org/sample_files/SampleE.pdf

!1. NCTM Problems of the Week. You need to be an NCTM member to access these problems at https://www.nctm.org/pows/