



WILLIAM & MARY

CHARTERED 1693

TEACHING PROBLEM SOLVING: LOOKING BEYOND THE ANSWERS

Sam Rhodes

The Tidewater Team

William & Mary School of Education



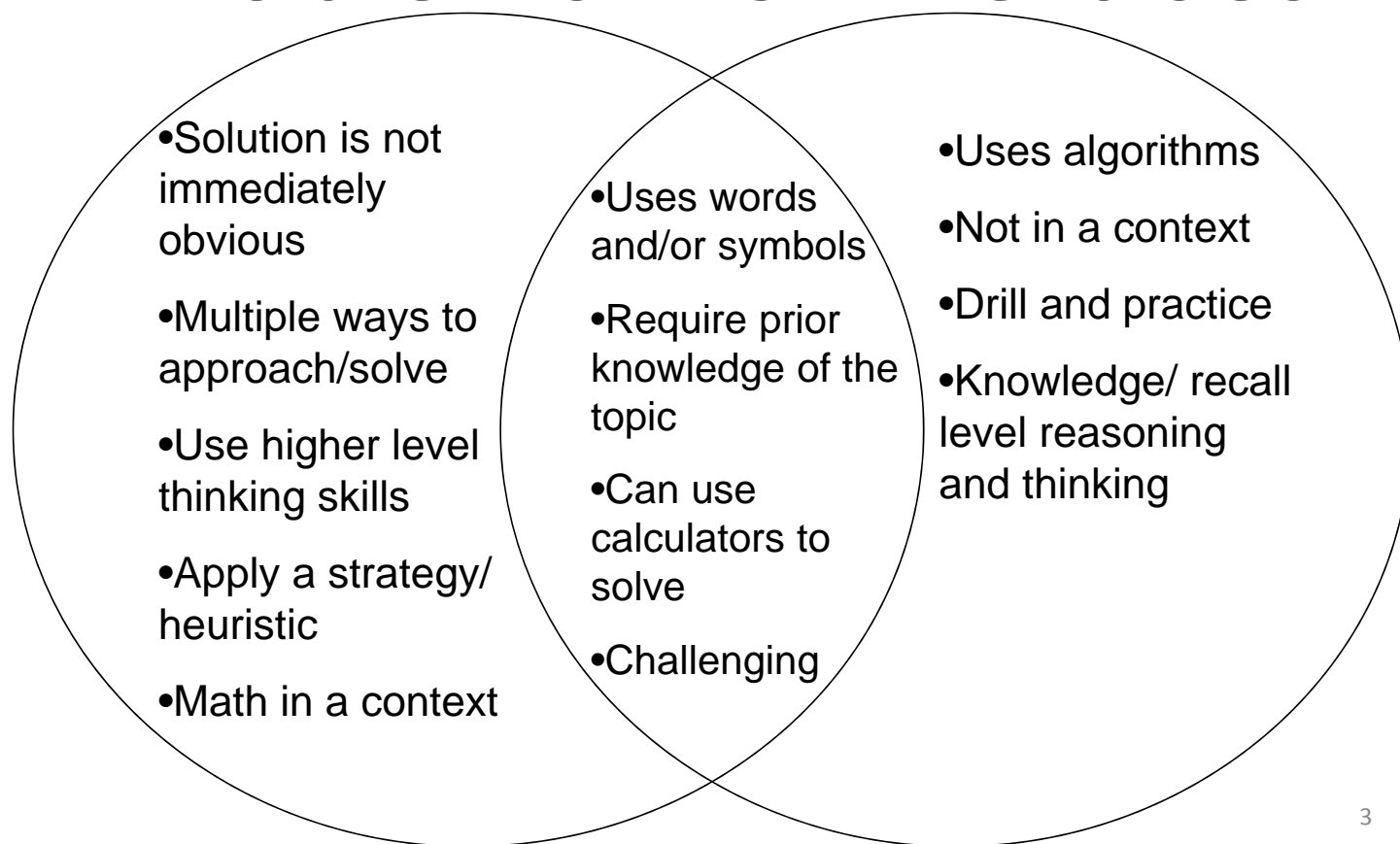
WELCOME/ILO

1. Understand the importance of problem solving instruction.
2. Understand why students struggle with problem solving
3. Learn several strategies for effectively teaching problem solving



WHAT IS PROBLEM SOLVING?

Problems VS Exercises





WILLIAM & MARY

CHARTERED 1693

TEACHING STUDENTS PROBLEM SOLVING IS LIKE...





SHARE OUT

Would anyone like to share out either what they said, or what somebody else said that resonated with them?



 **LiveSlides** web content

To view

Download the add-in.

liveslides.com/download

Start the presentation.



WELCOME

*The difference between school
and life?*

*In school, you're taught a lesson
and then given a test.*

*In life, you're given a test that
teaches you a lesson.*

Tom Bodett, humorist and author

BELIEFS ABOUT TEACHING MATH

| Unproductive beliefs | Productive beliefs |
|--|--|
| <p>Mathematics learning should focus on practicing procedures and memorizing basic number combinations.</p> | <p>Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.</p> |
| <p>Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.</p> | <p>All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.</p> |
| <p>Students can learn to apply mathematics only after they have mastered the basic skills.</p> | <p>Students can learn mathematics through exploring and solving contextual and mathematical problems.</p> |



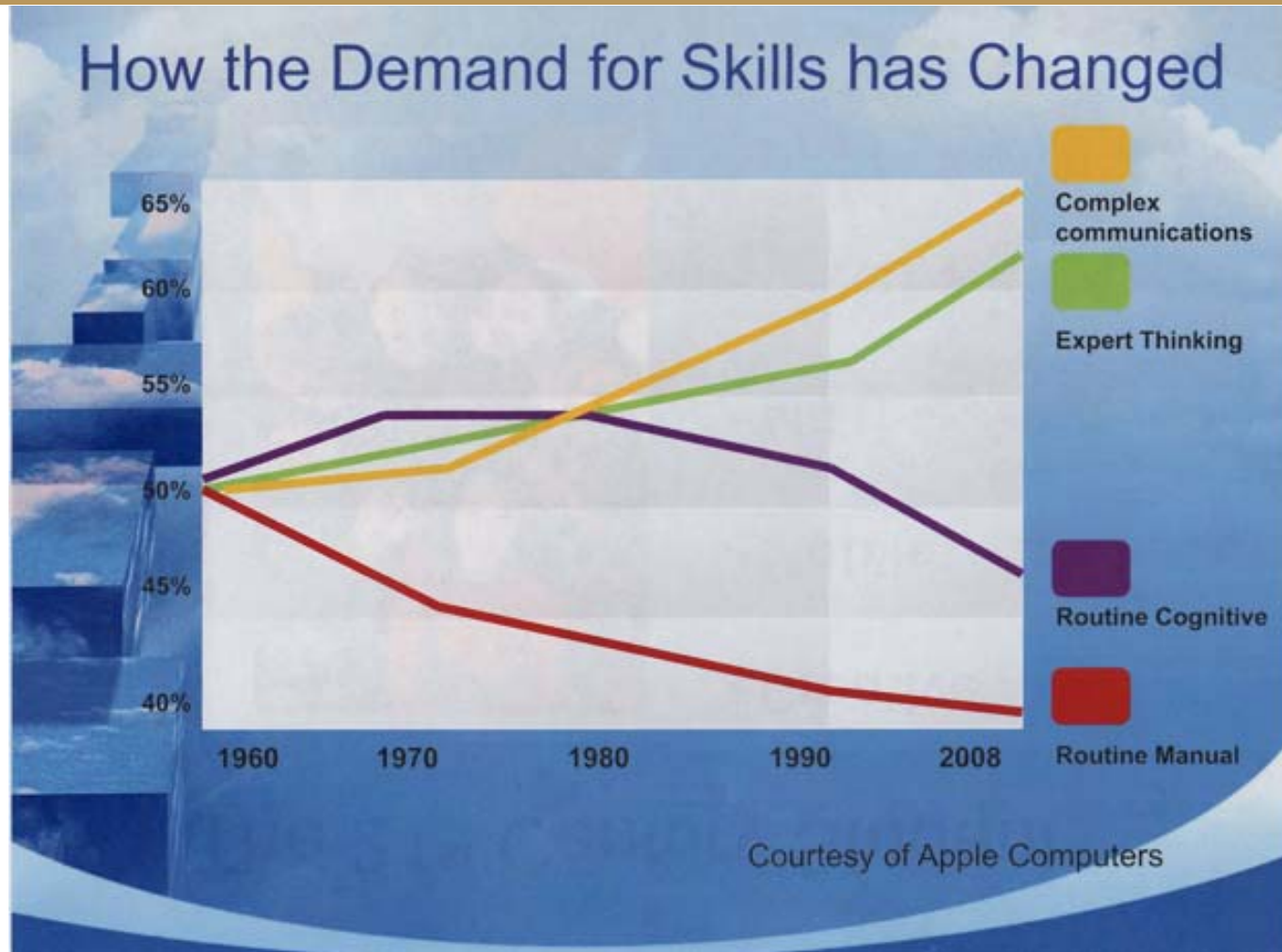
BELIEFS ABOUT TEACHING MATH

| Unproductive beliefs | Productive beliefs |
|--|---|
| <p>The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.</p> | <p>The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.</p> |
| <p>The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.</p> | <p>The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.</p> |

BELIEFS ABOUT TEACHING MATH

| Unproductive beliefs | Productive beliefs |
|--|--|
| <p>An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.</p> | <p>An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.</p> |

BELIEFS ABOUT TEACHING MATH





WHAT IS PROBLEM SOLVING?

“Mathematical tasks that have the potential to provide intellectual challenges for enhancing students' mathematical understanding and development” (Cai & Lester, 2010, p.1).

The process of applying previously acquired knowledge to new and unfamiliar situations (NCTM)



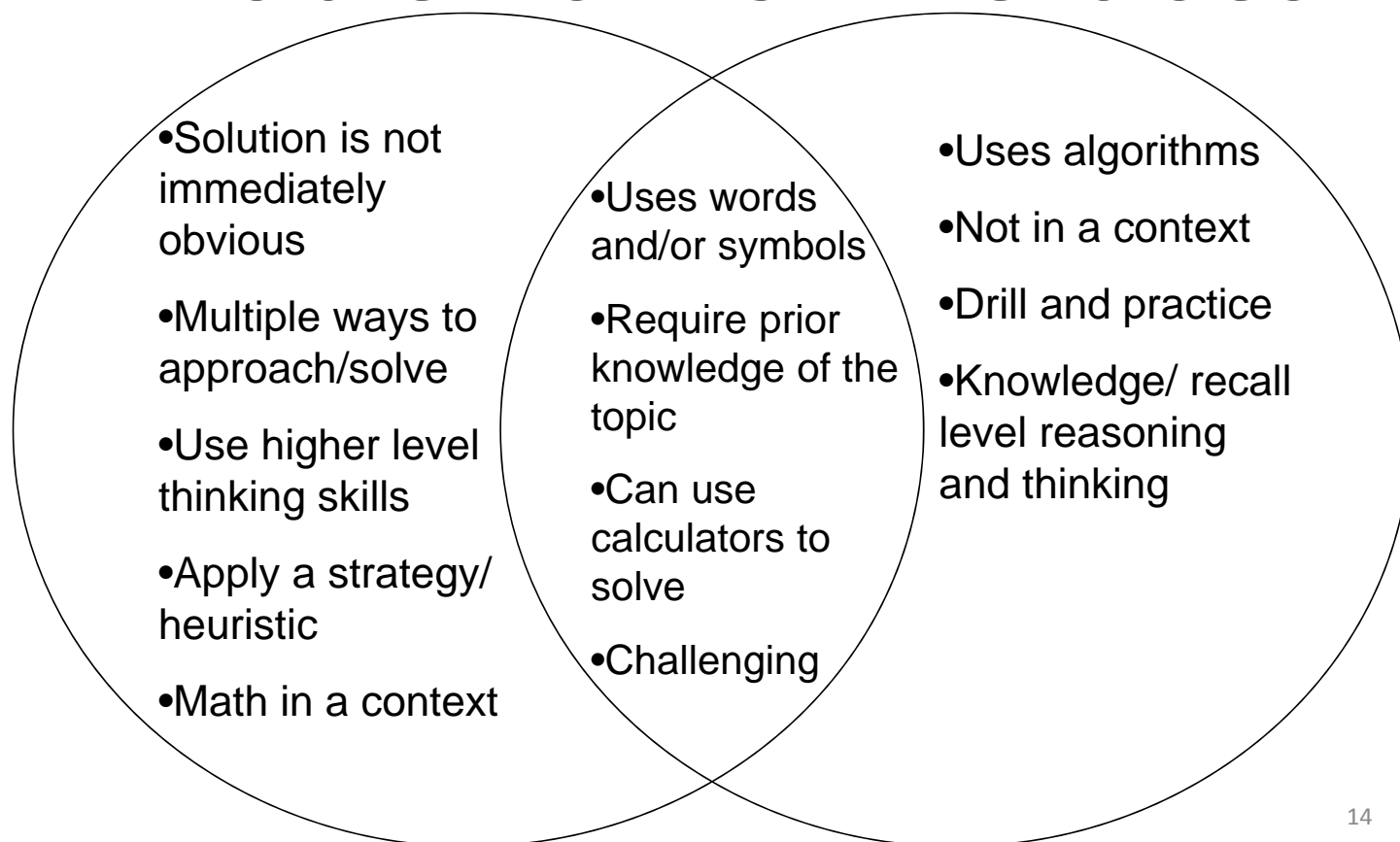
IMPLICATIONS

- Teach *through* problem solving
- Teach students to reflect and help them develop metacognitive awareness



WHAT IS PROBLEM SOLVING?

Problems VS Exercises





WHAT IS PROBLEM SOLVING?

Joe bought 3 packs of soda that had 12 cans of soda in each pack. How many cans of soda did he have in all?



WHAT IS PROBLEM SOLVING?

Wne bocmew **3** vcnxljhqe aslvnasv
asdvnas **12 cans** asl;dfj sadlfk asdf
ew. **How many cans of soda did he
have in all?**



WHAT IS PROBLEM SOLVING?

Brayden went to the store and bought Egg Nog for \$4, Cheese for \$6 and Crackers for \$3. If he paid with a \$20 bill, how much change did he receive?



WHAT IS PROBLEM SOLVING?

Lsajdf asdlkfj asdf ;ladsjf adfkl laks
df a;sdlkf **\$4**, asdf adf **\$6** alk
L;kj. Awer fa;f. **\$3**. nvc wewer wern
wl \$20 bill, **how much change did
he receive?**



KEY WORDS?

- What messages are we sending to students when we reduce problem solving to keywords?



WHAT IS PROBLEM SOLVING?

Joe ate $\frac{1}{3}$ of a pizza, and Kevin ate $\frac{2}{5}$ of the same pizza. Then, Jenna ate $\frac{1}{2}$ of the remaining pizza. How much of the pizza is left?



A STUDENTS' PROBLEM

A 48-inch steel rod is cut into two pieces. If the length of one piece is $\frac{1}{5}$ the length of the other piece, how many inches long is the shorter piece?

IDK??

What mistake did the student make?

What does the student know? What do they not know?



A STUDENTS' PROBLEM

A 48-inch steel rod is cut into two pieces. If the length of one piece is $\frac{1}{5}$ the length of the other piece, how many inches long is the shorter piece?

10 inches

What mistake did the student make?

What does the student know? What do they not know?



A STUDENTS' PROBLEM

A 48-inch steel rod is cut into two pieces. If the length of one piece is $\frac{1}{5}$ the length of the other piece, how many inches long is the shorter piece?

240 inches

What mistake did the student make?

What does the student know? What do they not know?



POLYA'S (1945) 4-STEP PROCESS

1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Look back



STEP 1: UNDERSTAND THE PROBLEM

With your tables, brainstorm what you would look for in the understand section.



STEP 1: UNDERSTAND THE PROBLEM

- Reread the problem
- What are you trying to find out?
- What facts do you have?
- What is the problem asking?
- What do you know that is not stated in the problem?
- What questions do you have about the situation?



STEP 1: UNDERSTAND THE PROBLEM

- Draw a picture to help you understand the problem.
- Try a specific case to help you learn about it.
- Is it possible to solve the problem?
- Is it a reasonable problem? Why or why not?
- Restate the problem in your own words



STEP 2: DEVISE A PLAN

How do we, as adults, form plans when we problem solve?

Have you ever been able to help a student solve a problem by simply asking them questions?



STEP 2: DEVISE A PLAN

- Do you know a related problem?
- Think of a familiar problem where you have to answer a similar question.
- Here is another problem related to yours. Could you use it to help you?
- Try to solve a simpler problem.
- What strategies (heuristics) do you know?



STEP 2: DEVISE A PLAN

- How can you organize the information given?
- What plan of attack will you use?
- Can you develop a pattern?
- Guess a solution or a first step.
- Can you write an equation?
- Display examples or cases in a chart or table.



WHAT ARE HEURISTICS?

Heuristics are any strategy that we use to help us solve a problem. Common ones include:

- Act it out
- Work backwards
- Guess and check
- Make a table
- Compute
- Look for patterns
- Write an equation
- Use drawings
- Consider a simpler case
- Simplify
- Use a formula



INQUIRY TRAINING/QFT

- Train students to ask their own questions
- Train students to evaluate the questions they are asking
- Teach students to recognize and leverage open and close ended questions



INQUIRY TRAINING/QFT

- Train students to ask their own questions
- Train students to evaluate the questions they are asking
- Teach students to recognize and leverage open and close ended questions

INQUIRY TRAINING/QFT

I had a strip of paper that I cut into 5 equal pieces. I then took 3 of those pieces and used magic to put them back together. The strip below represents those three pieces. Can you re-create the original paper?





INQUIRY TRAINING/QFT

- What yes/no questions do you think students might ask?
- What is the value of close ended questions?
- How can students use these questions in problem solving?



INQUIRY TRAINING/QFT

- What open-ended questions do you think students might ask?
- What is the value of open-ended questions?
- How can students use these questions in problem solving?



BREAK

10 minute break!



Image from: http://www.clipartpanda.com/clipart_images/break-time-free-clipart-58027541



STEP 3: CARRY OUT THE PLAN

- Use the strategy/heuristic you identified in the “Devise a Plan” phase
- Sometimes the original strategy you begin you leads to another
- Work the problem/solve



STEP 4: LOOK BACK

- Did you answer the question(s) asked?
- Is your answer in the correct unit?
- Does your answer seem reasonable?
- Describe your solution process.
- Are there extensions/generalizations you can make?
- Could you have solved the same problem another way?



STEP 4: LOOK BACK

- Could you have solved the same problem another way?
- Could you use the same method for another problem?
- What problems are similar to this one?
- Compare your solution to your guess or estimate.



STEP 4: LOOK BACK

- What are the advantages & disadvantages of your method?
- What were the pitfalls in this problem?
- Verbalize the stumbling blocks you overcame to solve this problem.
- What solution strategies were most helpful in solving the problem?



DEBRIEFING PROBLEMS

- Have students share their methods
- Value the process more than the solution
- Occasionally allow students time (potentially over night to brainstorm ideas/strategies)
- Use problem solving journaling

YOUR TURN!

The locker problem!





YOUR TURN!

A bowl holds a lot of pennies. The pennies can be divided into equal shares among 3, 4, 5, 6, 7, or 8 people with no pennies left over each time. What is the smallest number of pennies that can be in the bowl?



THINKING ABOUT YOUR OWN CLASSROOM...

- How could you introduce this process to your students?
- How can you develop their understanding of the process?
- How can you organize this process?
- What are the roadblocks you anticipate?



REFERENCES

Cai, J., & Lester, F. (2010). Why is Teaching with Problem Solving Important to Student Learning? *National Council of Teachers of Mathematics*, 13(12), 1–6. [https://doi.org/10.1016/S2213-8587\(14\)70016-6](https://doi.org/10.1016/S2213-8587(14)70016-6)

National Council of Teachers of Mathematics (NCTM). (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: NCTM.